# 2.4 The Dot Product of Two Vectors, the Length of a Vector, and the Angle Between Two Vectors

## THE DOT PRODUCT OF TWO VECTORS

The length of a vector or the angle between two vectors and can be found using the dot product.

The dot product of vectors and

is a scalar (real number) and is defined to be

Since and are real numbers, you can see that the dot product is itself a real number and not a vector.

To compute the dot product of the vectors and , we compute

Example (1)

Since the dot product is a scalar, it follows the properties of real numbers.

**PROPERTIES OF THE DOT PRODUCT**

1. , the dot product is commutative
2. , the dot product distributes over vector addition
3. , the dot product with the zero vector, is the scalar 0.

Compute the dot product ,

Example (2)

where , , and .

## THE LENGTH OF A VECTOR

The length (magnitude) of a vector you know is given by . The length can also be found using the dot product. If we dot a vector with itself, we get

By Vector Property 4, . This gives .

Taking the square root of each side produces

Which is the length of the vector .

The dot product of a vector with itself gives the length of the vector.

Use the dot product to find the length of the vector .

Example (3)

In this case, and

Using , we get

The length of the vector is units.

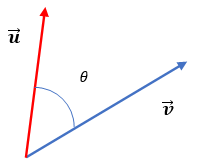
## THE ANGLE BETWEEN TWO VECTORS

The dot product and elementary trigonometry can be used to find the angle between two vectors.

If is the smallest nonnegative angle between two non-zero vectors and , then

cos or

where and and



Find the angle between the vectors and .

Example (4)

Using , we get

We conclude that the angle between these two vectors is close to 94.4°.

## USING TECHNOLOGY

We can use technology to find the angle between two vectors.

Go to www.wolframalpha.com.

To find the angle between the vectors and , enter angle between the vectors <5, -3> and <2, 4> in the entry field. Wolframalpha tells you what it thinks you entered, then tells you its answer. In this case, , rounded to one decimal place.

**Image of a calculation result of an angle between two vectors using WolframAlpha.**

## EXAMPLES

1. Find the dot product of the vectors and .

ANS:

1. Find the dot product of the vectors and .

ANS:

1. Find the length of the vector .

ANS:

4.Find the length of the vector .

ANS:

5.Find the angle between the vectors and .

ANS:

6.Find angle between the vectors and .

ANS:

## NOTE TO INSTRUCTOR

1. Perhaps begin by discussing the zero vector . This vector is represented by a single point. It has a length of measure 0.

1. Define the dot product of two vectors. Note that it is just a definition, and not derived. Then follow with an example.

Find the dot product of the vectors and .

1. Although it is developed at the beginning of the chapter, consider proving that

. It is instructive for students to see proofs as it helps to develop their logic.

Proof: We want to show for a vector , that .

For a vector

By Vector Property 4, . This gives .

Taking the square root of each side produces

1. Find the length of the vector .

Using with and ,

Make a conclusion. The length of the vector is 5 units.

1. Discuss the angle between two vectors and show an example of how to use the inverse cosine on the calculator.

Find the angle between the vectors and .

Example

Use with , and .

On the TI-84, input 2nd(–36/(2nd

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